Bsp $\quad f(x) = \cos(x)$

$$c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(x)\, e^{-ikx}\, dx$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2}\left(e^{ix} + e^{-ix}\right) e^{-ikx}\, dx$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2}\left(e^{i(1-k)x} + e^{i(-1-k)x}\right) dx$$

Fall $\quad k \notin \{-1, +1\}$:

$$c_k = \frac{1}{2\pi} \left[\frac{1}{2}\left(\frac{1}{i(1-k)} e^{i(1-k)x} + \frac{1}{i(-1-k)} e^{i(-1-k)x} \right) \right]_{-\pi}^{+\pi}$$

$$= 0 \quad - \quad \text{da } e^{ia\pi} = e^{i(-a)\pi} \text{ für } a \in \mathbb{Z}$$

Fall $\quad k = -1$

$$c_{-1} = \frac{1}{2\pi} \left[\frac{1}{2} \underbrace{\frac{1}{2i} e^{2ix}}_{\to 0} + \frac{1}{2} x \right]_{-\pi}^{+\pi} = \frac{1}{2}$$

Fall $\quad k = 1$

$$c_1 = \frac{1}{2\pi} \left[\frac{1}{2} x + \frac{1}{2} \frac{1}{-2i} e^{-2ix} \right]_{-\pi}^{+\pi} = \frac{1}{2}$$

$\Rightarrow$ für $f(x) = \cos(x)$

ist

$$\mathrm{Fourier}^{\mathbb{C}}_{f}(x) = \sum_{k=-\infty}^{+\infty} c_k e^{ikx}$$

nur $k \in \{-1,+1\}$

$$= c_{-1} e^{i\cdot(-1)x} + c_1 e^{i\cdot 1x}$$

$$= \frac{1}{2} e^{-ix} + \frac{1}{2} e^{ix}$$

$$= \cos(x)$$

$$\text{Fourier}_f^{C}(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$$

$$\overset{\text{E-dn}}{=} \sum_{k=-\infty}^{\infty} c_k \left(\cos(kx) + i \sin(kx) \right)$$

$\boxed{\text{cos gerade, sin ungerade}}$

$$= \underbrace{c_0}_{=a_0/2} \cos(0x)$$

$$+ \sum_{k=1}^{\infty} \overbrace{\left(c_k + c_{-k} \right)}^{=a_k} \cos(kx)$$

$$+ \sum_{k=1}^{\infty} \overbrace{\left(c_k - c_{-k} \right)}^{=b_k} i \sin(kx)$$