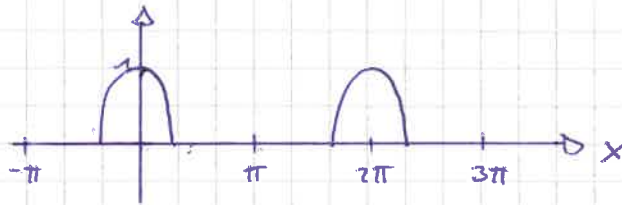


16) a)



b)

$$a_0 = \frac{2}{\pi} \int_0^1 (1-x^2) dx = \frac{2}{\pi} \left[x - \frac{1}{3}x^3 \right]_0^1 = \frac{2}{\pi} \left(1 - \frac{1}{3} \right) = \frac{4}{3\pi}$$

$k > 0$

$$a_k = \frac{2}{\pi} \int_0^1 (1-x^2) \cos(kx) dx = \frac{2}{\pi} \int_0^1 \cos(kx) dx - \frac{2}{\pi} \int_0^1 x^2 \cos(kx) dx$$

$$= \frac{2}{\pi} \left[\frac{1}{k} \sin(kx) \right]_0^1 - \frac{2}{\pi} \left(\left[x^2 \frac{1}{k} \sin(kx) \right]_0^1 - 2 \int_0^1 x \frac{1}{k} \sin(kx) dx \right)$$

$$= \frac{2}{k\pi} \sin(k) - \frac{2}{k\pi} \sin(k) + \frac{4}{k\pi} \left(\left[-x \frac{1}{k} \cos(kx) \right]_0^1 + \int_0^1 \frac{1}{k} \cos(kx) dx \right)$$

$$= \frac{4}{k\pi} \left(-\frac{1}{k} \cos(k) + \frac{1}{k^2} [\sin(kx)]_0^1 \right)$$

$$= \frac{-4}{\pi k^2} \cos(k) + \frac{4}{\pi k^3} \sin(k) = \frac{4 \sin(k) - 4k \cos(k)}{\pi k^3}$$

$$\text{Fourier}_f(x) = \frac{2}{3\pi} + \sum_{k=1}^{\infty} \frac{4 \sin(k) - 4k \cos(k)}{\pi k^3} \cos(kx)$$

c)

f stetig bei $x=0$ ✓

7.3.3

$$1 = f(0) = \text{Fourier}_f(0) = \frac{2}{3\pi} + \sum_{k=1}^{\infty} \frac{4 \sin(k) - 4k \cos(k)}{\pi k^3}$$

$$= \frac{2}{3\pi} + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(k) - k \cos(k)}{k^3}$$

$$\sum_{k=1}^{\infty} \frac{\sin(k) - k \cos(k)}{k^3} = \frac{\pi}{4} \frac{1 - \frac{2}{3\pi}}{4} = \frac{\pi - \frac{2}{3}}{4} = \frac{1}{4}\pi - \frac{1}{6}$$

d)

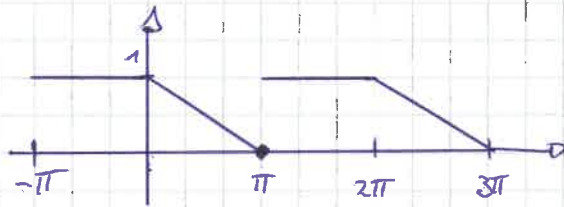
$$\int_{-1}^1 (1-x^2)^2 dx = \int_{-1}^1 1 - 2x^2 + x^4 dx = \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_{-1}^1 \quad \text{7.5.4}$$

$$= 1 - \frac{2}{3} + \frac{1}{5} + 1 - \frac{2}{3} + \frac{1}{5} = 2 - \frac{4}{3} + \frac{2}{5} = \frac{16}{15}$$

$$= \pi \frac{a_0^2}{2} + \pi \sum_{k=1}^{\infty} a_k^2 = \frac{8}{3}\pi + \pi \sum_{k=1}^{\infty} \frac{16 (\sin(k) - k \cos(k))^2}{\pi^2 k^6}$$

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a)



$$b) \quad c_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{2\pi} \left(\pi + \frac{\pi}{2} \right) = \frac{3}{4}$$

$$k \neq 0 \quad c_k = \frac{1}{2\pi} \int_{-\pi}^0 e^{-ikx} dx + \frac{1}{2\pi} \int_0^{\pi} \left(1 - \frac{1}{\pi}x\right) e^{-ikx} dx$$

$$= \frac{1}{2\pi} \left[\frac{1}{-ik} e^{-ikx} \right]_{-\pi}^0 + \frac{1}{2\pi} \int_0^{\pi} e^{-ikx} dx + \frac{1}{2\pi^2} \int_0^{\pi} x e^{-ikx} dx$$

$$= \frac{1}{2\pi} \left(\frac{1}{-ik} (1 - e^{ik\pi}) \right) + \frac{1}{2\pi} \left[\frac{1}{-ik} e^{-ikx} \right]_0^{\pi} - \frac{1}{2\pi^2} \left(\left[\frac{x e^{-ikx}}{-ik} \right]_0^{\pi} - \int_0^{\pi} \frac{1}{-ik} e^{-ikx} dx \right)$$

$$= \frac{1}{-2ik\pi} (1 - (-1)^k) + \frac{1}{-2ik\pi} ((-1)^k - 1) - \frac{1}{2\pi^2} \left(\frac{1}{-ik} \pi (-1)^k - \frac{1}{(-ik)^2} [e^{-ikx}]_0^{\pi} \right)$$

$$= \frac{1}{2ik\pi} (-1)^k + \frac{1}{-2k^2\pi^2} ((-1)^k - 1)$$

$$= \frac{1 - (-1)^k}{2k^2\pi^2} - i \frac{(-1)^k}{2k\pi}$$

$$\text{Fourier}_f^c(x) = \frac{3}{4} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \left(\frac{1 - (-1)^k}{2k^2\pi^2} - i \frac{(-1)^k}{2k\pi} \right) e^{ikx}$$

c)

$$a_0 = 2c_0 = \frac{3}{2}$$

$$k > 0 \quad a_k = 2 \operatorname{Re}(c_k) = \frac{1 - (-1)^k}{k^2\pi^2}$$

$$b_k = -2 \operatorname{Im}(c_k) = \frac{(-1)^k}{k\pi}$$

$$\text{Fourier}_f(x) = \frac{3}{4} + \sum_{k=1}^{\infty} \frac{1 - (-1)^k}{k^2\pi^2} \cos(kx) + \sum_{k=1}^{\infty} \frac{(-1)^k}{k\pi} \sin(kx)$$

7.6.3

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a)



b)

$$a_0 = \frac{2}{\pi} \int_0^{\pi/2} (1 - \frac{2}{\pi}x) dx = \frac{2}{\pi} \frac{\pi}{2} - \frac{4}{\pi^2} \left[\frac{1}{2}x^2 \right]_0^{\pi/2}$$

$$= 1 - \frac{2}{\pi^2} \frac{\pi^2}{4} = 1 - \frac{1}{2} = \frac{1}{2}$$

 $k > 0$

$$a_k = \frac{2}{\pi} \int_0^{\pi/2} (1 - \frac{2}{\pi}x) \cos(kx) dx$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \cos(kx) dx - \frac{4}{\pi^2} \int_0^{\pi/2} x \cos(kx) dx$$

$$= \frac{2}{\pi} \left[\frac{1}{k} \sin(kx) \right]_0^{\pi/2} - \frac{4}{\pi^2} \left(\left[\frac{1}{k} x \sin(kx) \right]_0^{\pi/2} - \frac{1}{k} \int_0^{\pi/2} \sin(kx) dx \right)$$

$$= \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) - \frac{4}{\pi^2} \left(\frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) + \frac{4}{k\pi^2} \left[-\frac{1}{k} \cos(kx) \right]_0^{\pi/2} \right)$$

$$= -\frac{4}{k^2\pi^2} \left(\cos\left(\frac{k\pi}{2}\right) - 1 \right)$$

$$\text{Fourier}_f(x) = \frac{1}{4} - \sum_{k=1}^{\infty} \frac{4}{k^2\pi^2} \left(\cos\left(\frac{k\pi}{2}\right) - 1 \right) \cos(kx)$$

c)

$$\text{Fourier}_g(x) = + \sum_{k=1}^{\infty} \frac{4}{k^2\pi^2} \left(\cos\left(\frac{k\pi}{2}\right) - 1 \right) \sin(kx)$$

↗ 7.4.7

d)

$$\text{Fourier}_g\left(\frac{\pi}{2}\right) = \frac{\lim_{x \rightarrow \frac{\pi}{2}^-} g(x) + \lim_{x \rightarrow \frac{\pi}{2}^+} g(x)}{2}$$

↗ 7.3.3

$$= \frac{\lim_{x \rightarrow \frac{\pi}{2}^-} f'(x) + \lim_{x \rightarrow \frac{\pi}{2}^+} f'(x)}{2}$$

$$= \frac{\frac{-2}{\pi} + 0}{2} = \frac{-2}{4} = -\frac{1}{\pi}$$

da $\text{Fourier}_g(x) = f'(x)$ wo f diffbar