Bem $z \cdot \bar{z} = (a+bi)(a-bi)$

$= a^2 - (bi)^2 = a^2 + b^2$

$= |z|^2$

Bem $z = a+bi$

$w = c+di$

$\bar{z} \cdot \bar{w} = (a-bi)(c-di)$

$= (ac-bd) - (ad+bc)i$

$z \cdot w = (a+bi)(c+di)$

$= (ac-bd) + (ad+bc)i$

$\overline{z \cdot w} = (ac-bd) - (ad+bc)i$

$=$

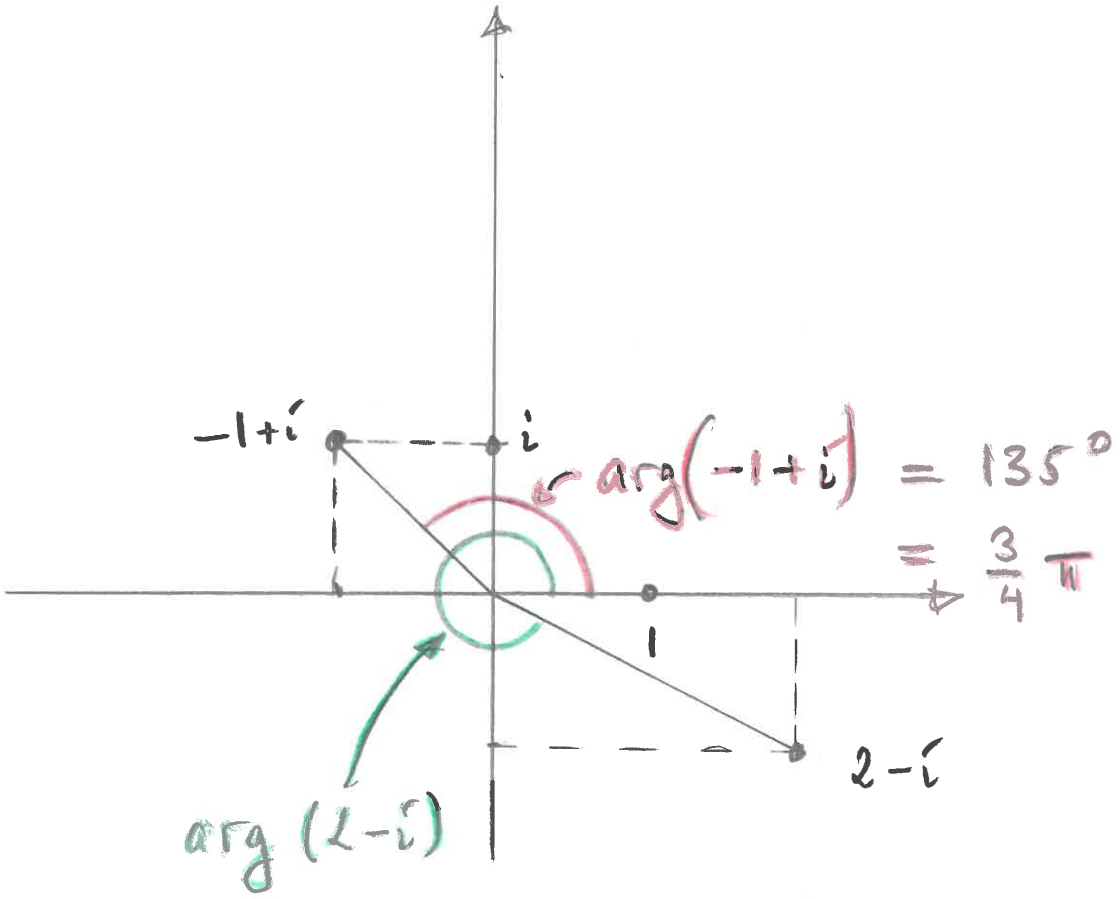
Bsp $z = 3+4i$

$\bar{z} = 3-4i$

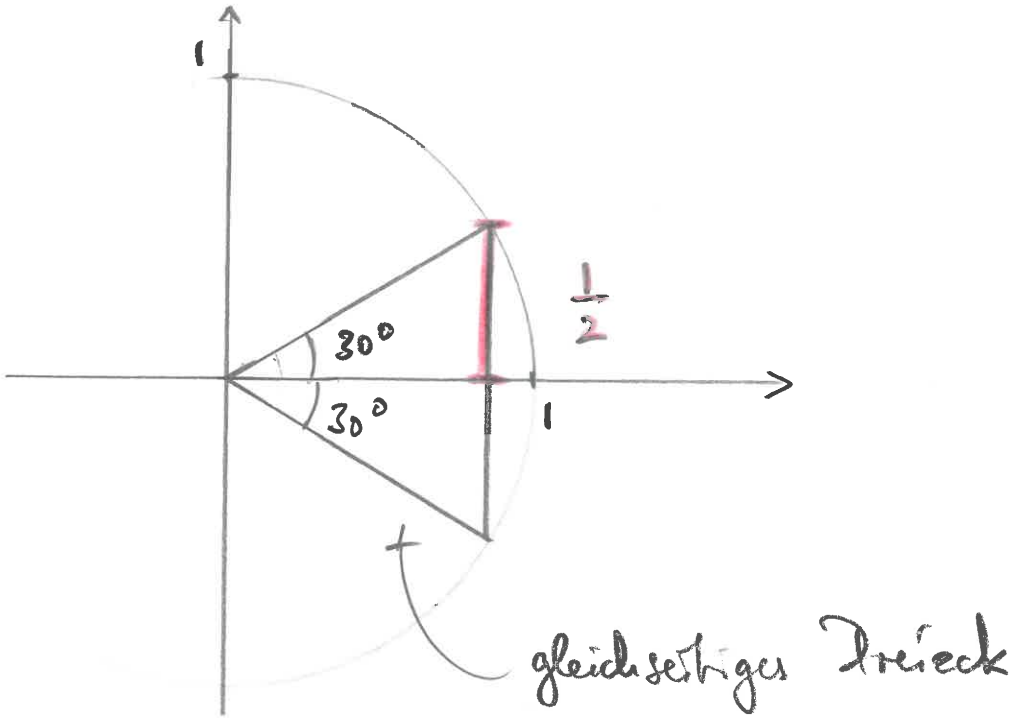
$|z| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

Bsp $|-2| = |-2+0i| = \sqrt{(-2)^2+0^2} = \sqrt{4} = 2$

Bsp



Bsp $\sin\left(\frac{\pi}{6}\right) = \sin(30°) = \frac{1}{2}$



Also $\cos\left(\frac{\pi}{6}\right) = \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{2}\sqrt{3}$

Also $\sin\left(\frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{6}\right) \cdot \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) \cdot \sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}$

$= \frac{1}{2}\sqrt{3}$

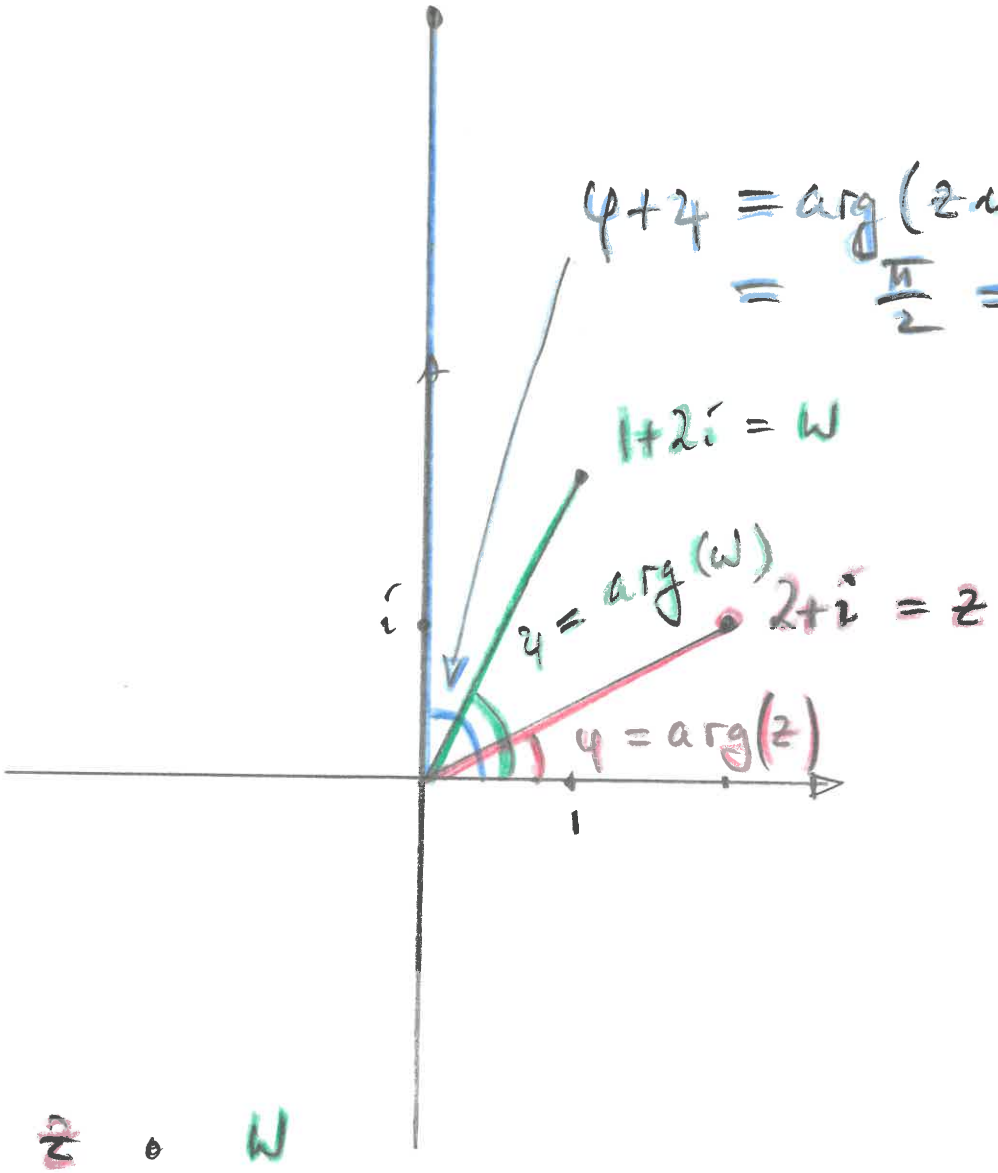
Dies folgt auch mit

$\sin\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{6}\right) = \frac{1}{2}\sqrt{3}$

$z \cdot w = 5i$

Bsp



$z \cdot w$

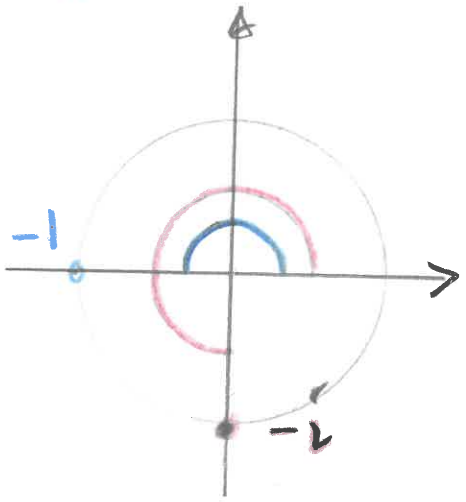
$= (2+i) \cdot (1+2i)$

$= 2 + i + 4i + 2i^2$

$= 5i$

"Bei Multiplikation addieren sich die Winkel"

Bsp $\arg(-i) = \frac{3\pi}{2}$



$$\arg\left((-i)\cdot(-i)\right)$$

$$= \underbrace{\arg(-i) + \arg(-i)}_{= \frac{3}{2}\pi + \frac{3}{2}\pi = 3\pi \in [2\pi, 4\pi[} - 2\pi$$

$$= 3\pi - 2\pi = \pi$$

Tatsächlich:

$$\arg\left((-i)\cdot(-i)\right) = \arg(-1) = \pi$$