<u>Bsp</u> Seien

$$A := \begin{pmatrix} -1 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \in \mathbb{F}_3^{4\times 6}$$

$$b := \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \in \mathbb{F}_3^{4\times 1}$$

Wir wollen

$$\{ x \in \mathbb{F}_3^{6\times 1} : Ax = b \}$$

bestimmen.

Wir formen um:

$$(A \mid b) = \ldots$$

15.12.2020 -2

$$= \begin{array}{c} \left(\begin{array}{cccccc|c} -1 & -1 & -1 & -1 & 1 & -1 & -1 \\ \boxed{1} & 1 & 0 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 & 0 & 0 & -1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}\right) \end{array} \begin{array}{c} +1 \\ \\ +1 \\ -1 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} \\ +1 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} -1 \\ -1 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

in Zeilenstufenform

(2) (4) (5)

Also wird für das inhomogene lineare Gleichungssystem:

$$\{x \in \mathbb{F}_3^{6\times 1} : Ax = b\}$$

$$= \underbrace{\begin{array}{c} \\ 2 \\ \\ 4 \\ 5 \\ \\ \end{array}\begin{pmatrix} 1 \\ \boxed{0} \\ 0 \\ \boxed{0} \\ \boxed{0} \\ 0 \end{pmatrix}}_{x_0} + \left\langle \underbrace{\begin{pmatrix} -1 \\ \boxed{1} \\ 0 \\ \boxed{0} \\ \boxed{0} \\ 0 \end{pmatrix}}_{x_1}, \underbrace{\begin{pmatrix} -1 \\ \boxed{0} \\ 0 \\ \boxed{1} \\ \boxed{0} \\ 0 \end{pmatrix}}_{x_2}, \underbrace{\begin{pmatrix} -1 \\ \boxed{0} \\ -1 \\ \boxed{0} \\ \boxed{1} \\ 0 \end{pmatrix}}_{x_3} \right\rangle_{\mathbb{F}_3}$$

Und für das zugehörige homogene lineare Gleichungssystem:

$$\{x \in \mathbb{F}_3^{6\times 1} : Ax = 0\}$$

$$= \left\langle \underbrace{\begin{pmatrix} -1 \\ \boxed{1} \\ 0 \\ \boxed{0} \\ \boxed{0} \\ 0 \end{pmatrix}}_{x_1}, \underbrace{\begin{pmatrix} -1 \\ \boxed{0} \\ 0 \\ \boxed{1} \\ \boxed{0} \\ 0 \end{pmatrix}}_{x_2}, \underbrace{\begin{pmatrix} -1 \\ \boxed{0} \\ -1 \\ \boxed{0} \\ \boxed{1} \\ 0 \end{pmatrix}}_{x_3} \right\rangle_{\mathbb{F}_3}$$

15.12.2020
-4

Probe für die Lösungsmenge für das inhomogene lineare Gleichungssystem:

$$A x_0 = b$$

$$A x_1 = 0$$

$$A x_2 = 0$$

$$A x_3 = 0$$

Probe für die Lösungsmenge des zugehörigen homogenen linearen Gleichungssystem:

nur $A x_1 = 0$, $A x_2 = 0$, $A x_3 = 0$.

Was wir nun schon wissen.

Bsp

Sei $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \end{pmatrix} \in \mathbb{F}_3^{4 \times 4}$

Wollen, so existent, die inverse Matrix A^{-1} bestimmen

$$(A \mid E_4) = \begin{array}{c} \left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \end{array} \begin{array}{c} \\ \\ -1 \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{c} \\ \\ -1 \\ +1 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & \boxed{1} & 1 & 0 & 1 & 0 & 1 \end{array}\right) \begin{array}{c} -1 \\ -1 \\ +1 \\ \\ \end{array}$$

$\rightsquigarrow \ldots$

$$\ldots \rightsquigarrow \begin{array}{c} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 & 1 \end{array}\right) \end{array} \begin{array}{l} \\ \\ \\ \cdot(-1) \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 & -1 \end{array}\right) \begin{array}{l} +1 \\ +1 \\ -1 \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 & -1 \end{array}\right)$$

$$\underbrace{}_{E_4} \qquad \underbrace{}_{A^{-1}}$$

$$\Rightarrow \quad A^{-1} = \begin{pmatrix} -1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 0 & -1 & -1 \end{pmatrix}$$

Probe: $A \cdot A^{-1} = E_4$

(oder $A^{-1} \cdot A = E_4$)