Bsp Sei $A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \in \mathbb{F}_3^{4\times 4}$

Wir suchen eine invertierbare Matrix $S \in \mathbb{F}_3^{4\times 4}$ mit $S^{-1}AS = J$ in Jordanscher Normalform.

$$\chi_A(X) = \det \begin{pmatrix} 1-X & 0 & 1 & 1 \\ -1 & -X & 1 & 0 \\ 1 & 0 & 1-X & 1 \\ 1 & 0 & 1 & 1-X \end{pmatrix}$$

$$= (-X) \det \begin{pmatrix} 1-X & 1 & 1 \\ 1 & 1-X & 1 \\ 1 & 1 & 1-X \end{pmatrix}$$

$$= (-X) \det \begin{pmatrix} -X & 1 & 0 \\ X & 1-X & X \\ 0 & 1 & -X \end{pmatrix}$$

$$= -X^3 \det \begin{pmatrix} -1 & 1 & 0 \\ 1 & 1-X & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} \\ +1 \\ +1 \end{matrix}$$

$$= -X^3 \det \begin{pmatrix} -1 & 1 & 0 \\ 0 & -X & 0 \\ 0 & 1 & -1 \end{pmatrix} = X^3 \det \begin{pmatrix} -X & 0 \\ 1 & -1 \end{pmatrix} = X^4.$$

Also ist

$\lambda_1 = 0$ der einzige

Eigenwert von A,

mit $aV_A(0) = 4$.

$$A_{(1)} = A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

Schrittweise ergänzte Basis von $H_A(0)$:

$\mathrm{Kern}(A_{(1)})$:

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \mathrm{Kern}(A_{(1)}) = \left\langle \underbrace{\begin{pmatrix} 0 \\ \boxed{1} \\ 0 \\ \boxed{0} \end{pmatrix}}_{x_{1,1}}, \underbrace{\begin{pmatrix} 1 \\ \boxed{0} \\ 1 \\ \boxed{1} \end{pmatrix}}_{x_{1,2}} \right\rangle$$

$$A' := \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

aus Zeilenstufenform

$$A_{(1)}^2 \rightsquigarrow A' \cdot A_{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\operatorname{Kern}\left(A_{(1)}^2\right)$:

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ \underbrace{0}_{\text{neu}} & 0 & \underbrace{0}_{\text{neu}} & 0 & 0 \end{array}\right)$$

$$\Rightarrow \operatorname{Kern}\left(A_{(1)}^2\right) = \left\langle \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{x_{1,1}}, \underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}}_{x_{1,2}}, \underbrace{\begin{pmatrix} \boxed{1} \\ \boxed{0} \\ \boxed{0} \\ \boxed{0} \end{pmatrix}}_{x_{2,1}}, \underbrace{\begin{pmatrix} \boxed{0} \\ \boxed{0} \\ \boxed{1} \\ \boxed{0} \end{pmatrix}}_{x_{2,2}} \right\rangle$$

$$\dim \operatorname{Kern}\left(A_{(1)}^2\right) = 4 = aV_A(0)$$

$\Rightarrow$ schrittweises Ergänzen beenden

$$H_A(0) = \mathrm{Kern}\left(A_{(1)}^2\right)$$

Nun: Hauptvektorketten berechnen:

$$y_{2,1} := x_{2,1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$A_{(1)} y_{2,1} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \underbrace{\left(\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right)}_{}$$

Hauptvektorkette

$$y_{2,2} := x_{2,2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$A_{(1)} y_{2,2} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \underbrace{\left(\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right)}_{}$$

Hauptvektorkette

Auswahl des leeren Tupels aus $(x_{1,1}, x_{1,2})$.

Somit : gilt

$$S := \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

wird $$S^{-1}AS = \begin{pmatrix} \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} \end{pmatrix}$$

$$=: J$$

Probe: $$A \cdot S = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\parallel$$

$$S \cdot J = \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} \end{pmatrix}$$

Paßt.

Bsp

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \in \mathbb{C}^{6\times 6}$$

Suchen: invertierbare Matrix $S \in \mathbb{C}^{6\times 6}$ mit $S^{-1}AS = J$ in Jordanscher Normalform.

$$\chi_A(X) = \det \begin{pmatrix} -X & 1 & 0 & 0 & 0 & 0 \\ -1 & -X & 1 & 0 & 0 & 0 \\ 0 & 0 & -X & 1 & 0 & 0 \\ 0 & 0 & -1 & -X & 0 & 0 \\ 0 & 0 & 0 & 0 & -X & 1 \\ 0 & 0 & 0 & 0 & -1 & -X \end{pmatrix}$$

$$= \det\begin{pmatrix} -X & 1 \\ -1 & -X \end{pmatrix} \cdot \det\begin{pmatrix} -X & 1 \\ -1 & -X \end{pmatrix} \cdot \det\begin{pmatrix} -X & 1 \\ -1 & -X \end{pmatrix}$$

$$= (X^2+1)^3 = (X-i)^3 \cdot (X+i)^3$$

Also haben wir die Eigenwerte

$$\lambda_1 = i \quad \text{mit} \quad aV_A(i) = 3$$

$$\lambda_2 = -i \quad \text{mit} \quad aV_A(-i) = 3$$

Zu $\lambda_1 = i$:

$$A_{(1)} = \begin{pmatrix} -i & 1 & 0 & 0 & 0 & 0 \\ -1 & -i & 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 & 0 \\ 0 & 0 & -1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 1 \\ 0 & 0 & 0 & 0 & -1 & -i \end{pmatrix}$$

Berechnen nun schrittweise ergänzte Basis des Hauptraums.

$\operatorname{Kern}\left(A_{(1)}\right)$:

$$\left(\begin{array}{cccccc|c} -i & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -i & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -i & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccccc|c} 1 & i & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{c|c} \underbrace{\begin{matrix} 1 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i \end{matrix}}_{=: A'} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \end{array}\right)$$

$$\Rightarrow \operatorname{Kern} A_{(1)} = \left\langle \underbrace{\begin{pmatrix} -i \\ \boxed{1} \\ 0 \\ 0 \\ 0 \\ \boxed{0} \end{pmatrix}}_{x_{1,1}}, \underbrace{\begin{pmatrix} 0 \\ \boxed{0} \\ 0 \\ 0 \\ -i \\ \boxed{1} \end{pmatrix}}_{x_{1,2}} \right\rangle$$

$$A_{(1)}^2 \rightsquigarrow A' \cdot A_{(1)}$$

$$= \left(\begin{array}{cccccc|c} -2i & 2 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2i & 2 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & i & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & i & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & 0 \end{array}\right)$$

neu

$$\Rightarrow \operatorname{Kern}\left(A_{(1)}^2\right) = \left\langle \underbrace{\begin{pmatrix} -i \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{x_{1,1}}, \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -i \\ 1 \end{pmatrix}}_{x_{1,2}}, \underbrace{\begin{pmatrix} -i/2 \\ \boxed{0} \\ -i \\ \boxed{1} \\ 0 \\ \boxed{0} \end{pmatrix}}_{x_{2,1}} \right\rangle$$

$$\dim \operatorname{Kern}(A_{(i)}^2) = 3 = aV_A(i)$$

$$\Rightarrow \quad H_A(i) = \operatorname{Kern}(A_{(i)}^2)$$

Nun: Hauptvektorketten berechnen.

$$y_{2,1} := x_{2,1} = \begin{pmatrix} -i/2 \\ 0 \\ -i \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$A_{(i)} y_{2,1} = \begin{pmatrix} -i & 1 & 0 & 0 & 0 & 0 \\ -1 & -i & 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 & 0 \\ 0 & 0 & -1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 1 \\ 0 & 0 & 0 & 0 & -1 & -i \end{pmatrix} \begin{pmatrix} -i/2 \\ 0 \\ -i \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 \\ -i/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = -\frac{i}{2} x_{1,1}$$

$$\Rightarrow \quad y_{1,1} := x_{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -i \\ 1 \end{pmatrix}$$

Hauptvektorketten

$$\left(A_{(1)} y_{2,1},\ y_{2,1}\right) = \left(\begin{pmatrix}-\frac{1}{2}\\ -\frac{i}{2}\\ 0\\ 0\\ 0\\ 0\end{pmatrix} \begin{pmatrix}-\frac{i}{2}\\ 0\\ -i\\ 1\\ 0\\ 0\end{pmatrix}\right)$$

$$\left(y_{1,1}\right) = \left(\begin{pmatrix}0\\ 0\\ 0\\ 0\\ -i\\ 1\end{pmatrix}\right)$$

Zu $\lambda_2 = -i$:

$$A_{(2)} = \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0\\ -1 & i & i & 0 & 0 & 0\\ 0 & 0 & i & 1 & 0 & 0\\ 0 & 0 & -1 & i & 0 & 0\\ 0 & 0 & 0 & 0 & -i & 1\\ 0 & 0 & 0 & 0 & -1 & -i \end{pmatrix}$$

Berechnen nun schrittweise ergänzte Basis des Hauptraums.

$\mathrm{Kern}\left(A_{(2)}\right)$:

$$\left(\begin{array}{cccccc|c} i & 1 & 0 & 0 & 0 & 0 & 0\\ -1 & i & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & i & 1 & 0 & 0 & 0\\ 0 & 0 & -1 & i & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & i & 1 & 0\\ 0 & 0 & 0 & 0 & -1 & i & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccccc|c} 1 & -i & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & i & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & i & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & i & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -i & 0 \end{array}\right)$$

$$\underbrace{}_{=:\, A'}$$

$$\Rightarrow \operatorname{Ker}\left(A_{(2)}\right) = \left\langle \begin{pmatrix} i \\ \boxed{1} \\ 0 \\ 0 \\ 0 \\ \boxed{0} \end{pmatrix}, \begin{pmatrix} 0 \\ \boxed{0} \\ 0 \\ 0 \\ i \\ \boxed{1} \end{pmatrix} \right\rangle$$

$$A_{(2)}^2 \rightsquigarrow A' \cdot A_{(2)}$$

$$= \left(\begin{array}{cccccc|c} 2i & 2 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i & 2 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & -i & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccccc|c} 1 & -i & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & \underset{\text{neu}}{0} & 1 & -i & 0 \end{array}\right)$$

$$\Rightarrow \operatorname{Ker}\left(A_{(2)}^2\right) = \left\langle \underbrace{\begin{pmatrix} i \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{x_{1,1}}, \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 1 \end{pmatrix}}_{x_{1,2}}, \underbrace{\begin{pmatrix} i/2 \\ \boxed{0} \\ i \\ \boxed{1} \\ 0 \\ \boxed{0} \end{pmatrix}}_{x_{2,1}} \right\rangle$$

$$\dim \operatorname{Kern}\left(A_{(2)}^2\right) = 3 = aV_A(-i)$$

$$\Rightarrow H_A(-i) = \operatorname{Kern}\left(A_{(2)}^2\right)$$

Nun: Hauptvektorketten berechnen.

$$y_{2,1} := x_{2,1} = \begin{pmatrix} i/2 \\ 0 \\ i \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$A_{(2)} y_{2,1} = \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0 \\ -1 & i & 1 & 0 & 0 & 0 \\ 0 & 0 & i & 1 & 0 & 0 \\ 0 & 0 & -1 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & -1 & i \end{pmatrix} \begin{pmatrix} i/2 \\ 0 \\ i \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 \\ i/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{i}{2} x_{1,1}$$

$$\Rightarrow y_{1,1} := x_{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 1 \end{pmatrix}$$

Hauptvektorketten

$$\left(A_{(2)}\, y_{2,1},\ y_{2,1}\right) = \Big(\underbrace{\begin{pmatrix} -1/2 \\ i/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} i/2 \\ 0 \\ i \\ 1 \\ 0 \\ 0 \end{pmatrix}} \Big)$$

$$y_{1,1} = \Big(\underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 1 \end{pmatrix}} \Big)$$

Zusammensetzen:

Mit
$$S = \begin{pmatrix} -1/2 & -i/2 & 0 & -1/2 & i/2 & 0 \\ -i/2 & 0 & 0 & i/2 & 0 & 0 \\ 0 & -i & 0 & 0 & i & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -i & 0 & 0 & i \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

wird
$$S^{-1} A S = J = \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 1 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \end{pmatrix}$$

zur Jordanschen Normalform von A.

01.02.21-14

Probe :

$$A \cdot S = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -1/2 & -i/2 & 0 & -1/2 & i/2 & 0 \\ -i/2 & 0 & 0 & i/2 & 0 & 0 \\ 0 & -i & 0 & 0 & i & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -i & 0 & 0 & i \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -i/2 & 0 & 0 & i/2 & 0 & 0 \\ 1/2 & -i/2 & 0 & 1/2 & i/2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 & -i & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & i & 0 & 0 & -i \end{pmatrix}$$

$$S \cdot J = \begin{pmatrix} -1/2 & -i/2 & 0 & -1/2 & i/2 & 0 \\ -i/2 & 0 & 0 & i/2 & 0 & 0 \\ 0 & -i & 0 & 0 & i & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -i & 0 & 0 & i \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 1 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \end{pmatrix}$$

$$= \begin{pmatrix} -i/2 & 0 & 0 & i/2 & 0 & 0 \\ 1/2 & -i/2 & 0 & 1/2 & i/2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 & -i & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & i & 0 & 0 & -i \end{pmatrix}$$

=

Pa/Sr.