Beispiel für Integration direkt mit Hauptsatz

Wir sollen $\int_2^5 \frac{x^3+x^2+x}{x^2+1}\,dx$ berechnen.

Polynomdivision:

$$(x^3+x^2+x) = (x^2+1)\cdot(x+1) + (-1)$$

$$\begin{array}{l} x^3 \quad +x \\ \hline x^2 \\ x^2+1 \\ \hline -1 \end{array}$$

Also

$$\int \frac{x^3+x^2+x}{x^2+1}\,dx = \ldots$$

27.05.2018

$$\ldots = \int x+1 - \frac{1}{x^2+1}\, dx$$

$$= \int x+1\, dx - \int \frac{1}{x^2+1}\, dx$$

$$= \left[\frac{1}{2}x^2 + x\right] - \left[\arctan(x)\right]$$

$$= \left[\frac{1}{2}x^2 + x - \arctan(x)\right]$$

beachte: $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$

Somit wird

$$\int_2^5 \frac{x^3+x^2+x}{x^2+1}\, dx$$

$$= \left[\frac{1}{2}x^2 + x - \arctan(x)\right]_2^5$$

$$= \left(\frac{1}{2}5^2 + 5 - \arctan(5)\right)$$

$$- \left(\frac{1}{2}2^2 + 2 - \arctan(2)\right) = \ldots$$

$$\ldots = \frac{27}{2} - \arctan(5) + \arctan(2)$$

Bsp für Integration direkt nach Hauptsatz

$$\int_0^1 \sqrt{x}\, dx$$

$$= \int_0^1 x^{\frac{1}{2}}\, dx$$

$$= \left[\frac{1}{\frac{3}{2}} x^{\frac{3}{2}} \right]_0^1 = \frac{2}{3} \cdot \left[x^{\frac{3}{2}} \right]_0^1$$

$$= \frac{2}{3} \left(1^{\frac{3}{2}} - 0^{\frac{3}{2}} \right) = \frac{2}{3}$$

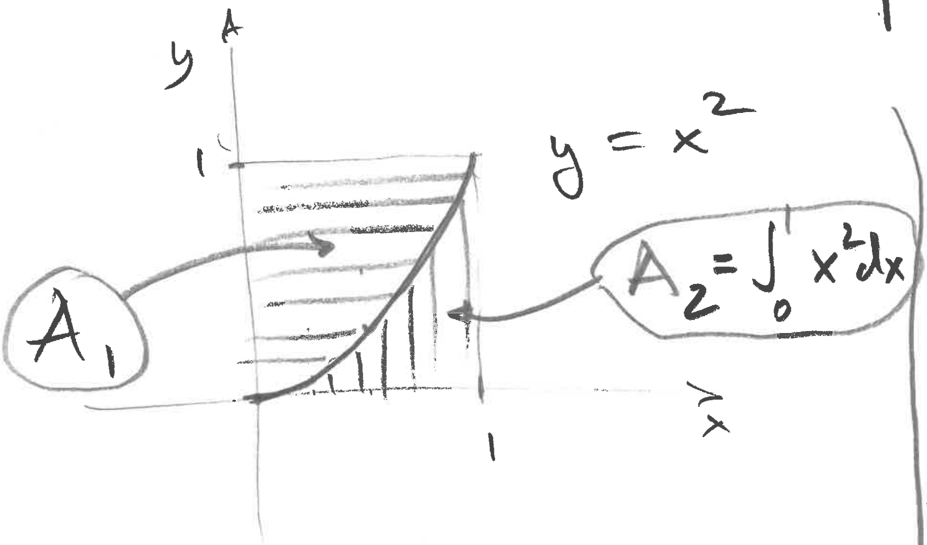
27.05.20 – 10

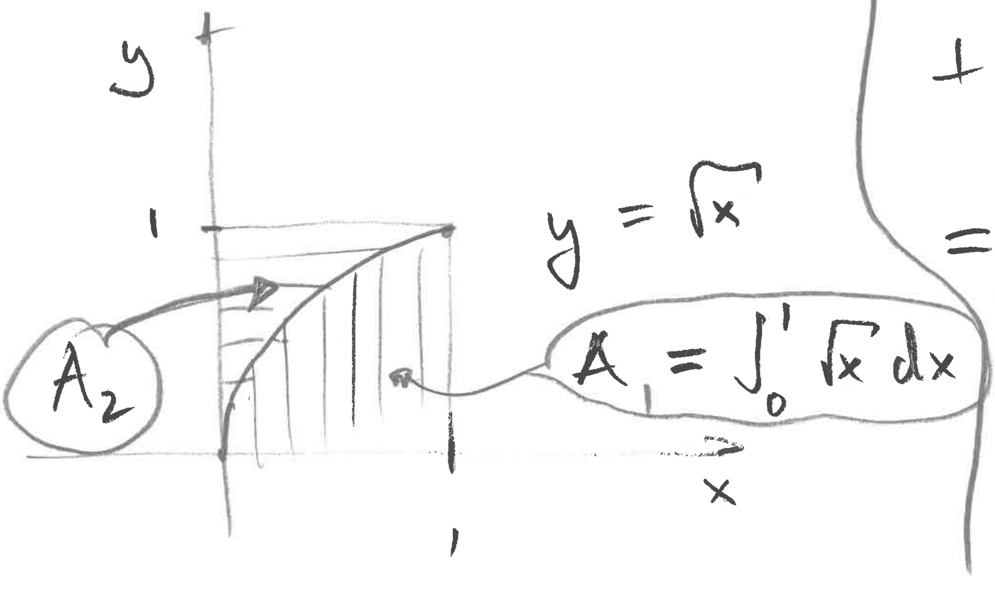
$$\int_0^1 x^2\,dx = \left[\frac{1}{3}x^3\right]_0^1$$

$$= \frac{1}{3}\left[x^3\right]_0^1$$

$$= \frac{1}{3}\left(1^3 - 0^3\right) = \frac{1}{3}$$

Graphisch:





$1 =$ (Flächeninhalt des Quadrats)

$= A_1 + A_2$

$= \int_0^1 \sqrt{x}\,dx + \int_0^1 x^2\,dx$

$= \frac{2}{3} + \frac{1}{3}$, das paßt