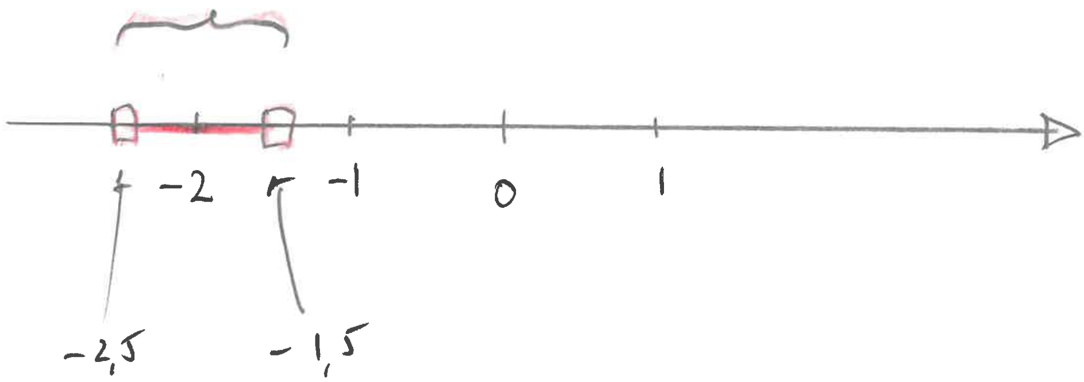
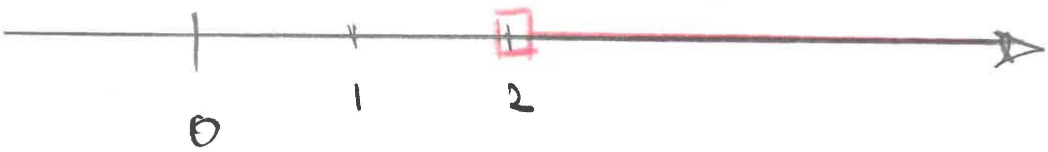
Bsp:

(1) $U_{\frac{1}{2}}(-2) =]-2{,}5 \,,\, -1{,}5[$

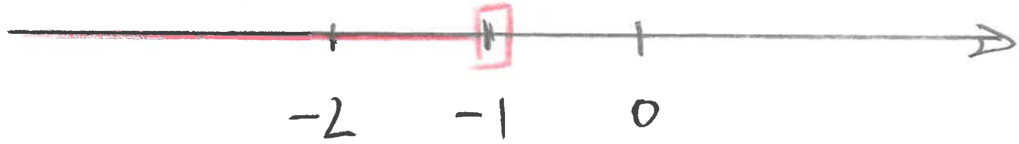


(2)

$U_2(+\infty)$

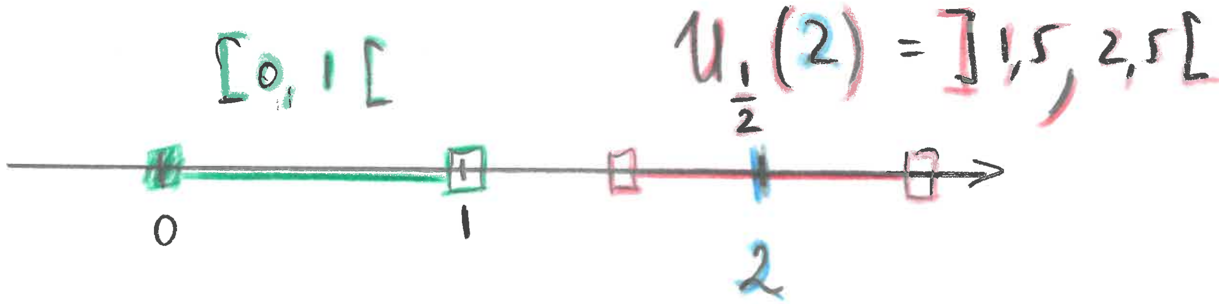


$U_1(-\infty)$



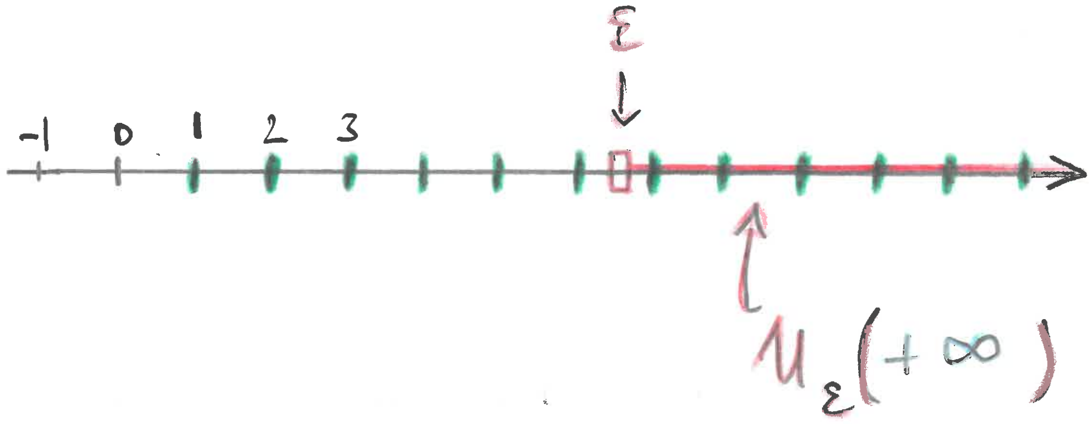
Bsp

(1) 2 hängt nicht an $[0,1[$:



z.B. ist $[0,1[\cap U_{\frac{1}{2}}(2) = \emptyset$

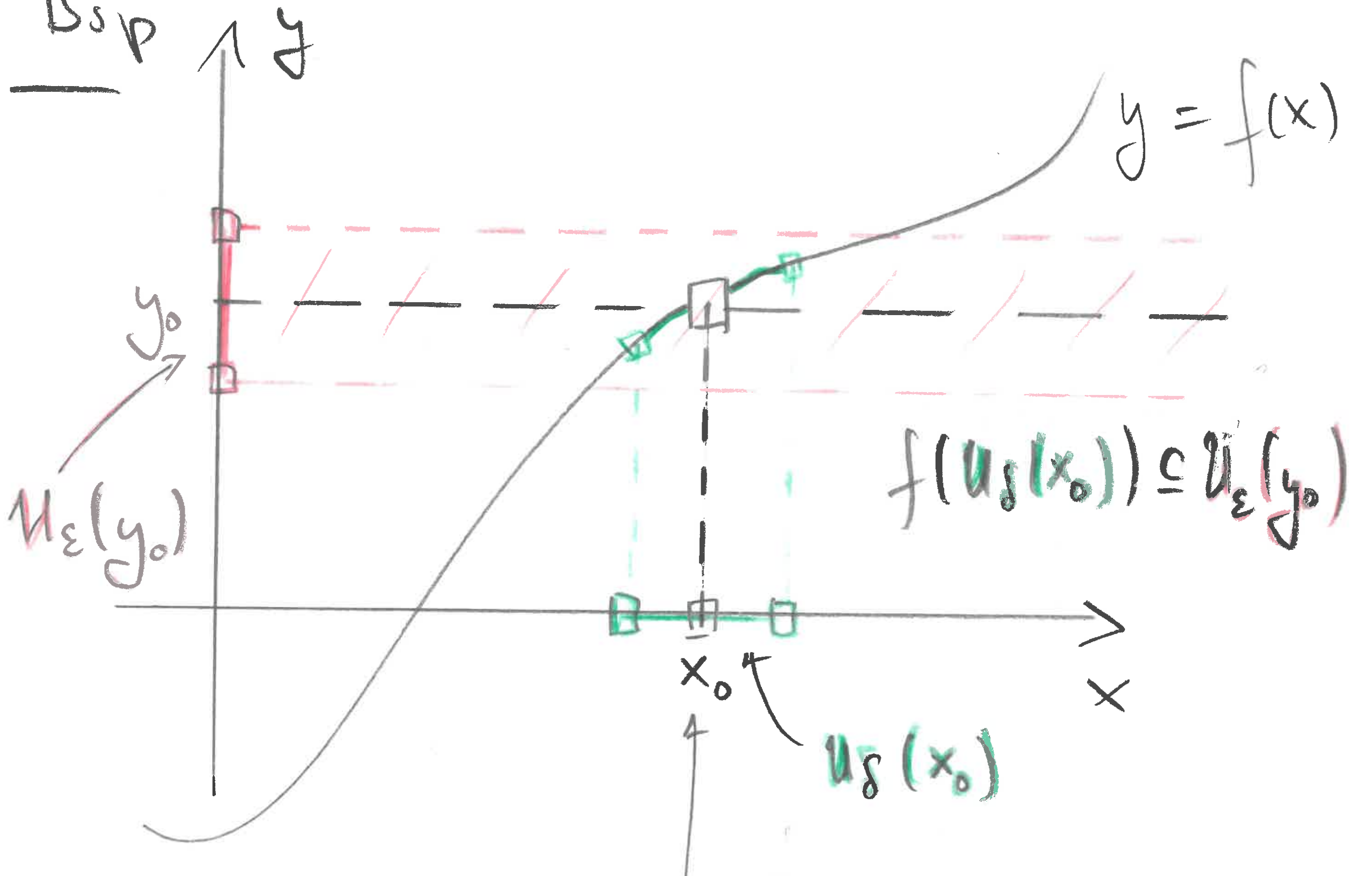
(2) $+\infty$ hängt an $\mathbb{Z}_{\geq 1}$:



Für jedes $\varepsilon \in \mathbb{R}_{>0}$

ist $\mathbb{Z}_{\geq 1} \cap U_\varepsilon(+\infty) \neq \emptyset$

Bsp

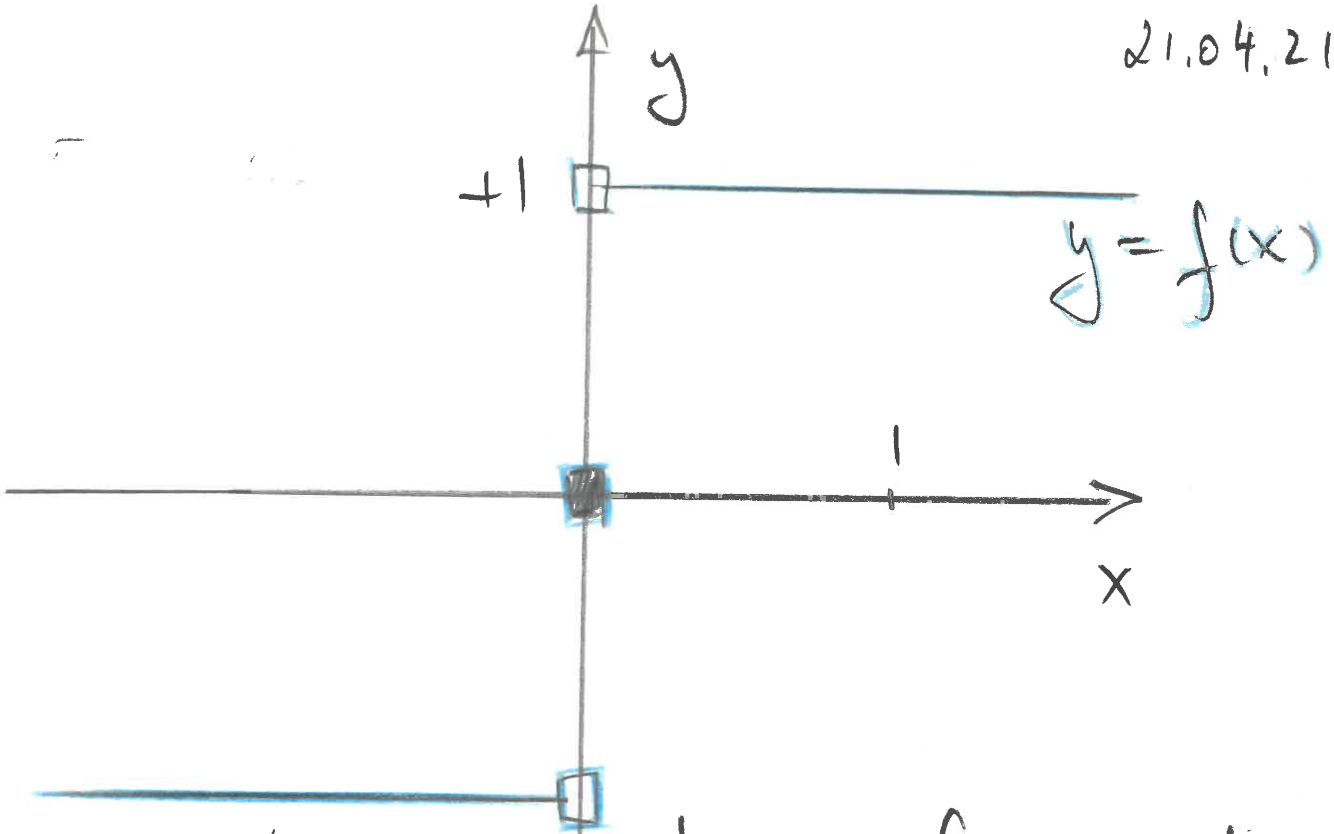


"Definitionslücke"
von f

Bsp Sei

$$f : \underbrace{\mathbb{R}}_{=D} \longrightarrow \mathbb{R}$$

$$x \longmapsto \begin{cases} +1 & \text{falls } x > 0 \\ 0 & \text{falls } x = 0 \\ -1 & \text{falls } x < 0 \end{cases}$$



Dann ist $y_0 = 0$ kein Grenzwert von f bei $x_0 = 0$. Denn z.B. ist:

$$f^{-1}\left(\mathcal{U}_{\frac{1}{2}}(0) \right)$$

$$= \{ x \in \mathbb{R} : f(x) \in \mathcal{U}_{\frac{1}{2}}(0) \}$$

$$= \{ x \in \mathbb{R} : -\tfrac{1}{2} < f(x) < +\tfrac{1}{2} \}$$

$$= \{ 0 \}$$

Und daher gibt es kein $\delta \in \mathbb{R}_{>0}$ mit $\mathcal{U}_\delta(0) =]-\delta, +\delta[\subseteq \{0\} = f^{-1}(\mathcal{U}_{\frac{1}{2}}(0))$.