Bsp. Sei $f : \mathbb{R}\setminus\{0\} \to \mathbb{R} : x \mapsto \frac{1}{x^2}$.

Wir wollen $f'(x)$ auf drei Weisen berechnen.

(1) $f(x) = x^{-2}$

$$f'(x) = -2\,x^{-2-1} = -\frac{2}{x^3}$$

(2) $f(x) = x^{-2} = x^{-1} \cdot x^{-1}$

$$f'(x) = \left(x^{-1}\right)' \cdot x^{-1} + x^{-1} \cdot \left(x^{-1}\right)'$$

$$= (-1) \cdot x^{-1-1} \cdot x^{-1} + x^{-1} \cdot (-1)\, x^{-1-1}$$

$$= -x^{-3} - x^{-3} = -\frac{2}{x^3}$$

(3) $$f(x) = \frac{1}{x^2}$$

$$f'(x) = \frac{(1)' \cdot x^2 - 1 \cdot (x^2)'}{(x^2)^2}$$

$$= \frac{0 \cdot x^2 - 1 \cdot 2x}{x^4}$$

$$= -\frac{2x}{x^4} = -\frac{2}{x^3}$$

Bsp Sei $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$;

$$x \longmapsto f(x) = \frac{1}{x^3+1}$$

$$\Rightarrow f'(x) = \frac{(1)'(x^3+1) - 1 \cdot (x^3+1)'}{(x^3+1)^2}$$

$$= \frac{-3x^2}{(x^3+1)^2}$$

$$= \ldots$$

$$\ldots = -3 \cdot \frac{x^2}{(x^3+1)^2}$$

$$\Rightarrow f''(x) = -3 \cdot \frac{(x^2)' \cdot (x^3+1)^2 - x^2 \cdot ((x^3+1)^2)'}{((x^3+1)^2)^2}$$

$$= -3 \cdot \frac{2x \cdot (x^3+1)^2 - x^2(x^6+2x^3+1)'}{(x^3+1)^4}$$

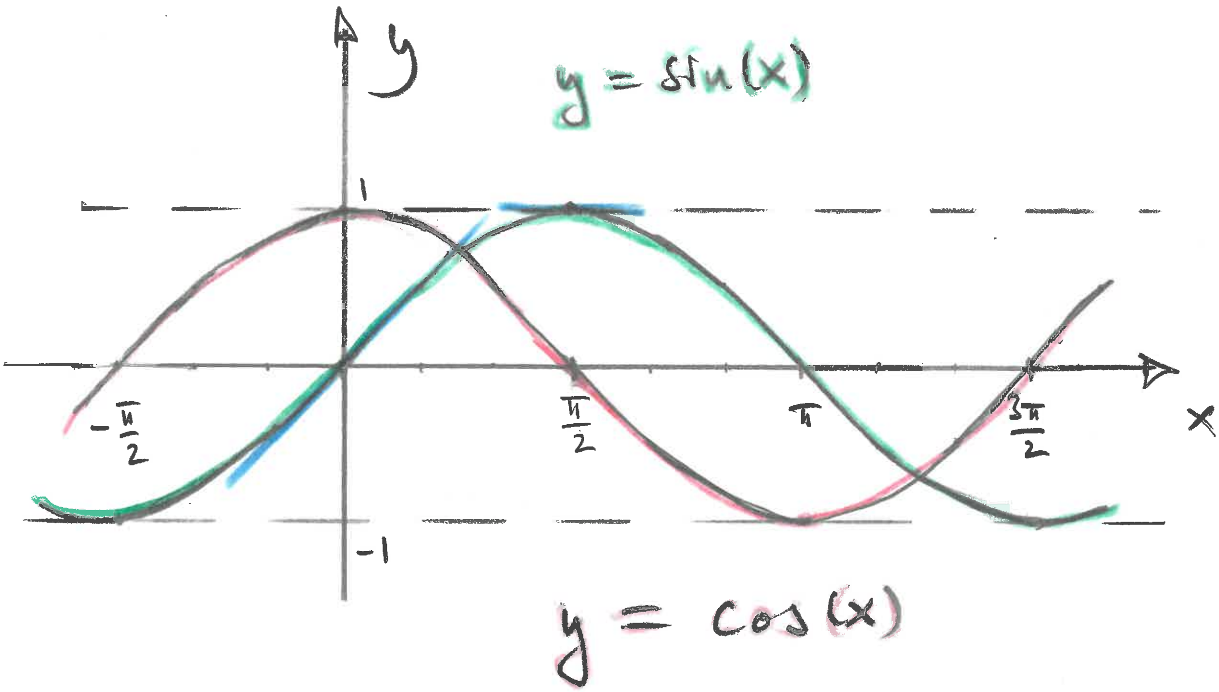
$$= -3 \cdot \frac{2x \cdot (x^3+1)^2 - x^2(6x^5+6x^2)}{(x^3+1)^4}$$

$$= -3 \cdot \frac{2x \cdot (x^3+1) - x^2 \cdot 6x^2}{(x^3+1)^3}$$

$$= -3 \cdot \frac{-4x^4 + 2x}{(x^3+1)^3}$$

$$= 6 \frac{2x^4 - x}{(x^3+1)^3}$$

Bsp



Bsp

$$f(x) = \sin(x^2)$$

$$f'(x) = \cos(x^2) \cdot 2x$$

$$f''(x) = -\sin(x^2) \cdot (2x)^2 + \cos(x^2) \cdot 2$$

$$f'''(x) = -\cos(x^2) \cdot (2x)^3 - \sin(x^2) \cdot 8x - \sin(x^2) \cdot 4x = \ldots$$

$$\ldots = -\cos(x^2) \cdot 8x^3$$

$$- \sin(x^2) \cdot 12x$$