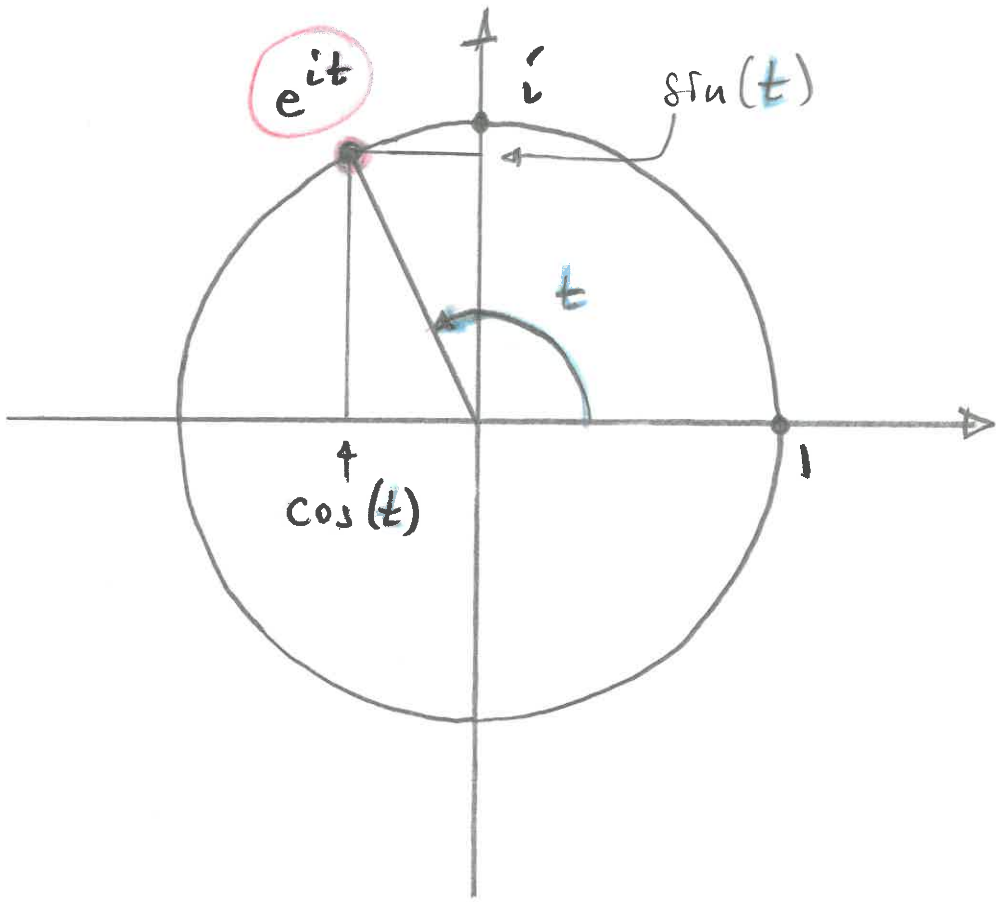
__Bsp__ Sei $t \in \mathbb{R}$.

Es ist $e^{it} = \cos(t) + i\sin(t)$



z.B.

$$e^{i\frac{\pi}{2}} = \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + i\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = i$$

$$e^{i\pi} = \underbrace{\cos(\pi)}_{-1} + i\underbrace{\sin(\pi)}_{0} = -1$$

$$e^{i\frac{3\pi}{2}} = \underbrace{\cos\left(\frac{3\pi}{2}\right)}_{0} + i\underbrace{\sin\left(\frac{3\pi}{2}\right)}_{-1} = -i$$

Bsp Es wird

$$e^{2+i\pi} = e^2 \cdot e^{i\pi} = -e^2 .$$

Bsp Es wird

$$\sin(x)^3 = \left(\frac{1}{2i}\left(e^{ix} - e^{-ix}\right)\right)^3$$

$$= \frac{1}{(2i)^3}\left(e^{ix} - e^{-ix}\right)^3$$

$$\overset{\text{Binomischer Lehrsatz}}{=} \frac{1}{-8i}\Big(\left(e^{ix}\right)^3 - 3\left(e^{ix}\right)^2 e^{-ix} + 3e^{ix}\left(e^{-ix}\right)^2 - \left(e^{-ix}\right)^3\Big)$$

$$= -\frac{1}{8i}\left(e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{-3ix}\right)$$

$$= -\frac{1}{8i}\left(\left(e^{3ix} - e^{-3ix}\right) - 3\left(e^{ix} - e^{-ix}\right)\right)$$

$$= -\frac{1}{8i}\left(2i\sin(3x) - 3\cdot 2i\sin(x)\right)$$

$$= -\frac{1}{4}\sin(3x) + \frac{3}{4}\sin(x)$$

<u>Bsp</u> (1) $\frac{d}{dx}\cosh(x)$

$$= \frac{d}{dx}\left(\frac{1}{2}\left(e^{x} + e^{-x}\right)\right)$$

$$= \frac{1}{2}\left(e^{x} - e^{-x}\right) = \sinh(x)$$

(2) $\frac{d}{dx} \sinh(x)$

$$= \frac{d}{dx}\left(\frac{1}{2}\left(e^{x} - e^{-x}\right)\right)$$

$$= \frac{1}{2}\left(e^{x} + e^{-x}\right) = \cosh(x)$$

<u>Bsp</u> Es ist

$$\log_{10}(1000) = \log_{10}(10^{3}) \overset{(5)}{=} 3$$