Bsp Wir wollen

$$u''' - u = 0$$

zur Anfangsbedingung

$$u(0) = 1$$

$$u'(0) = 0$$

$$u''(0) = 0$$

lösen.

Umschreiben!

$$y_1(x) := u(x)$$

$$y_2(x) := u'(x)$$

$$y_3(x) := u''(x)$$

Dann:

$$y_1' = y_2$$
$$y_2' = y_3$$
$$y_3' = u''' \overset{!}{=} \quad u = y_1$$

Also

$$y' = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}_{A} y \,, \quad y(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Longrightarrow \quad y(x) = \exp(Ax) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Siehe 14.07.21-1,
=
14.07.21-7

...

$$\dots \quad \frac{2e^{-\frac{x}{2}}}{3} \begin{pmatrix} \cos\left(\frac{x\sqrt{3}}{2}\right) & \cos\left(\frac{x\sqrt{3}}{2} - \frac{2\pi}{3}\right) & \cos\left(\frac{x\sqrt{3}}{2} + \frac{2\pi}{3}\right) \\ \cos\left(\frac{x\sqrt{3}}{2} + \frac{2\pi}{3}\right) & \cos\left(\frac{x\sqrt{3}}{2}\right) & \cos\left(\frac{x\sqrt{3}}{2} - \frac{2\pi}{3}\right) \\ \cos\left(\frac{x\sqrt{3}}{2} - \frac{2\pi}{3}\right) & \cos\left(\frac{x\sqrt{3}}{2} + \frac{2\pi}{3}\right) & \cos\left(\frac{x\sqrt{3}}{2}\right) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$+ \frac{e^x}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= 2\frac{e^{-\frac{x}{2}}}{3} \begin{pmatrix} \cos\left(\frac{x\sqrt{3}}{2}\right) \\ \cos\left(\frac{x\sqrt{3}}{2} + \frac{2\pi}{3}\right) \\ \cos\left(\frac{x\sqrt{3}}{2} - \frac{2\pi}{3}\right) \end{pmatrix} + \frac{e^x}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad u(x) = y_1(x)$$

$$= \frac{2}{3} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^x$$

Probe:

$$u'(x) = -\frac{1}{3} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) - \frac{1}{\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^{x}$$

$$\begin{aligned} u''(x) &= \frac{1}{6} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) \\ &\quad + \frac{1}{2\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) \\ &\quad + \frac{1}{2\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) \\ &\quad - \frac{1}{2} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^{x} \\ &= -\frac{1}{3} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) \\ &\quad + \frac{1}{\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^{x} \end{aligned}$$

$$u'''(x) = \frac{1}{6} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right)$$

$$+ \frac{1}{2\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right)$$

$$- \frac{1}{2\sqrt{3}} e^{-\frac{x}{2}} \sin\left(\frac{x\sqrt{3}}{2}\right)$$

$$+ \frac{1}{2} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^{x}$$

$$= \frac{2}{3} e^{-\frac{x}{2}} \cos\left(\frac{x\sqrt{3}}{2}\right) + \frac{1}{3} e^{x}$$

$$= u(x)$$

ferner: $u(0) = \frac{2}{3} + \frac{1}{3} = 1$

$$u'(0) = -\frac{1}{3} + 0 + \frac{1}{3} = 0$$

$$u''(0) = -\frac{1}{3} + 0 + \frac{1}{3} = 0$$

Pa/t